Math 45 SSM 2/e 2.8 Solving Linear Inequalities in One Variable
Day 2

Objectives
Day 1
1) Set-builder notation
2) Interval notation
3) Graphing intervals
Day 2
4) Solve linear inequalities.
5) Word Problems with Linear Inequalities

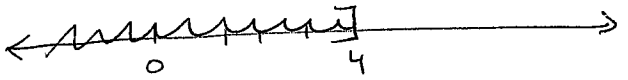
GOALS: 1) Solve linear inequalities for all values of x that are solutions
2) Write the solution set in interval notation
3) Graph the solution set
4) Recognize when inequality has no solution
5) Recognize when inequality has all real #s as solutions.

Linear inequalities are mostly like linear equations—
we isolate the variable using
- reverse order of operations
- the inverse of what we have
  - remove add w/ subtract
  - remove subtract w/ add
  - remove mult w/ divide
  - remove divide w/ mult
- do the same to both sides of the inequality.

There is ONE KEY RULE that is different:
When we multiply or divide by a negative number, we must reverse the direction of the inequality in the next step of work.

To see why this is, let's consider some examples:

(A) $x \leq 4$ means



test; $x=0$: $0 \leq 4$ true.
0 is shaded.

(B) Add 1 to both sides:

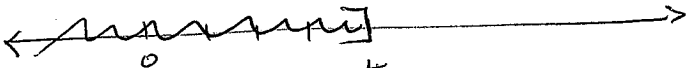
$x+1 \leq 4+1$

$x+1 \leq 5$

test $x=4$: $4+1 \leq 5$ true

test $x=0$: $0+1 \leq 5$ true.

test $x=5$: $5+1 \leq 5$ false



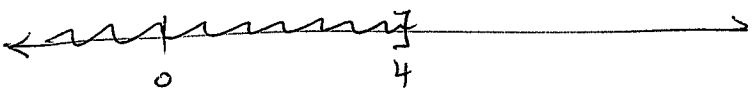
We get the same resulting set of x-values.

(C) Subtract 1 to both sides: (Same as add −1!)

$x-1 \leq 4-1$

$x-1 \leq 3$

same set.



(D) Multiply by 2 to both sides:

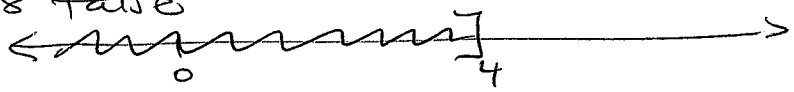
$2x \leq 4 \cdot 2$

$2x \leq 8$

test $x=0$: $2 \cdot 0 \leq 8$ true

test $x=4$: $2 \cdot 4 \leq 8$ true

test $x=5$ $2 \cdot 5 \leq 8$ false



(E) Multiply by −2 to both sides:

$-2x \leq 4 \cdot (-2)$

$-2x \leq -8$

test $x=0$: $-2(0) \leq -8$

$0 \leq -8$ false! Woah!

test $x=4$: $-2(4) \leq -8$ true

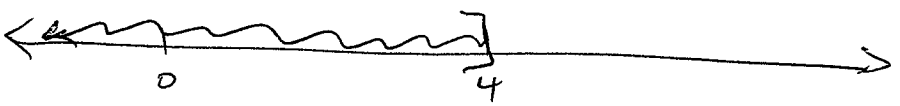
test $x=5$: $-2(5) \leq -8$

$-10 \leq -8$ true!

(F) Divide by 2

$\frac{x}{2} \leq \frac{4}{2}$

$\frac{x}{2} \leq 2$



(G) Divide by -2

$$\frac{x}{-2} \leq \frac{4}{-2}$$

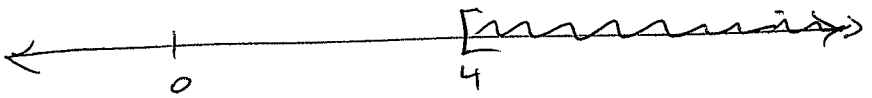
$$\frac{-x}{2} \leq -2$$

test $x=0$ $\quad \frac{-0}{2} \leq -2$ false

$x=4$ $\quad \frac{+4}{-2} \leq -2$ true

$x=5$ $\quad \frac{+5}{-2} \leq -2$

$-\frac{5}{2} \leq -2$ true.



VERY IMPORTANT:

We can add or subtract by any number except zero AND we can multiply both sides by any positive number or divide both sides by any positive number WITHOUT changing the solution.

BUT when we want to multiply or divide by a negative number we must change the direction of the inequality.

Examples

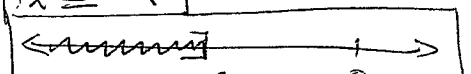
Solve the inequality. Graph the solution. Write solution in interval notation.

(1) $6 \geq x + 15$

$-15 \quad -15$ Subtract 15 both sides

$-9 \geq x$ rewrite – reverse all

Solve $x \leq -9$

Graph 

Interval $(-\infty, -9]$

(2) $-\frac{5}{8}x > \frac{25}{48}$

$-\frac{5}{8}x \cdot 48 > \frac{25}{48} \cdot 48$ clear fractions

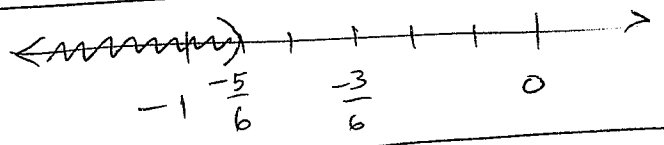
$-5x \cdot 6 > 25$

$-30x > 25$

② cont

$\frac{-30x}{-30} < \frac{25}{-30}$ { divide both sides by -30 { change $>$ to $<$

Solve $x < \frac{-5}{6}$ reduce

graph 

interval $(-\infty, -\frac{5}{6})$

Handwriting: Be very careful with b and 6!

③ $-5b + 2(b-1) \leq 6 - 5b - 1$

$-5b + 2b - 2 \leq 6 - 5b - 1$ dist

$-3b - 2 \leq 5 - 5b$ combine

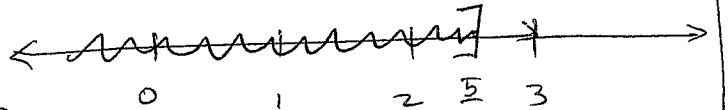
$+5b \quad +5b$

$2b - 2 \leq 5$ add $5b$ to both sides

$+2 \quad +2$ add 2 to both sides

$\frac{2b}{2} \leq \frac{5}{2}$ divide both sides by 2

Solve $b \leq \frac{5}{2}$

Graph 

Interval $(-\infty, \frac{5}{2}]$

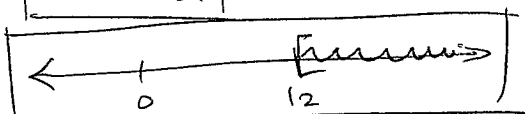
④ $\frac{x}{3} \geq 2 + \frac{x}{6}$

$6 \cdot \frac{x}{3} \geq 6 \cdot 2 + 6 \cdot \frac{x}{6}$ clear fractions — multiply each term by 6. = LCD

$2 \cdot x \geq 12 + x$ divide out first

$-x \quad -x$ subtract x both sides

$x \geq 12$



$[12, +\infty)$

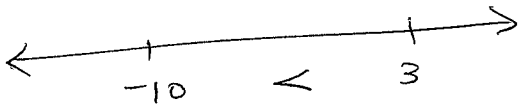
⑤ $5(x-2) < 3(x+1) + 2x$

$5x - 10 < 3x + 3 + 2x$ Distribute

$5x - 10 < 5x + 3$ Combine

$-5x \quad -5x$ subtract $5x$ both sides

$-10 < 3$



$-10 < 3$ is a true statement

Any x-value we substitute should work:

Explore: You do not need to do this on exam or PQ.

check: $x = 0$ $\quad 5(0-2) < 3(0+1) + 2(0)$

$5(-2) < 3(1) + 0$

$-10 < 3$ ✓

check: $x = -66$ $\quad 5(-66-2) < 3(-66+1) + 2(-66)$

$5(-68) < 3(-65) - 132$

$-340 < -195 - 132$

$-340 < -327$ ✓

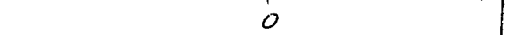
check: $x = 10$: $\quad 5(10-2) < 3(10+1) + 2(10)$

$5(8) < 3(11) + 2(10)$

$40 < 33 + 20$

$40 < 53$ ✓

Solution: all real numbers

graph: 

interval: $(-\infty, \infty)$

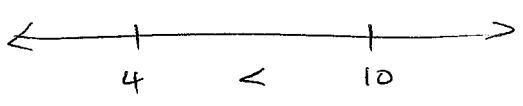
⑥ $2k - (k-4) \geq 3k + 10 - 2k$

$2k - k + 4 \geq 3k + 10 - 2k$ Distribute

$k + 4 \geq k + 10$ Combine

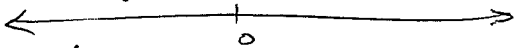
$-k \quad -k$ subtract k both sides

$4 \geq 10$



$4 \geq 10$ is a false statement.

Solution: empty set, no solution

graph: 

interval: $\emptyset$

(7) A student scored 72%, 74% and 78% on the first three exams. What interval of scores does the student need on exam #4 to have a B average?

average 4 values: $\frac{72+74+78+x}{4}$

B-average means 80% or higher.

average ≥ 80

$\frac{72+74+78+x}{4} \geq 80$ combine

$\frac{224+x}{4} \geq 80$ multiply by 4

$224+x \geq 80(4)$

$224+x \geq 320$

$-224 \quad -224$

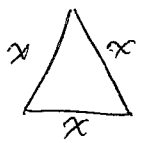
$x \geq 96$

Student must earn at least 96%.

Interval $[96, 100]$

(8) If the perimeter of an equilateral triangle is less than 15 feet, how long could each side be? Write result as interval.

perimeter $x+x+x$

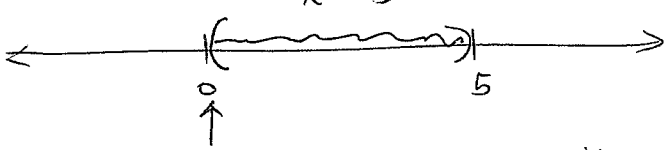


$= 3x$

less than 15: $3x < 15$

divide by 3: $x < \frac{15}{3}$

$x < 5$



can't be zero or negative length!

$(0, 5)$

Solve, graph, interval.

⑨ $\frac{3z-1}{4} + 1 \leq \frac{6z+5}{2} + 2$

$4 \cdot \frac{(3z-1)}{4} + 4 \cdot 1 \leq 4 \cdot \frac{(6z+5)}{2} + 2 \cdot 4$ clear frac LCD = 4 mult all terms

$3z - 1 + 4 \leq 2(6z+5) + 8$

$3z - 1 + 4 \leq 12z + 10 + 8$ distribute

$3z + 4 \leq 12z + 18$ combine

$-3z \quad -3z$

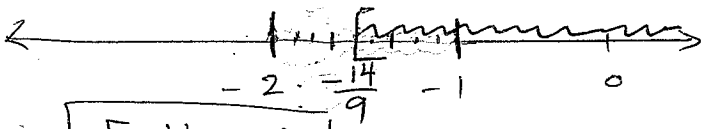
$4 \leq 9z + 18$ subtract 3z

$-18 \quad -18$

$\frac{-14}{9} \leq \frac{9z}{9}$ subtract 18

$\frac{-14}{9} \leq z$ divide by 9.

$\boxed{z \geq \frac{-14}{9}} = -1\frac{5}{9}$ rewrite by reversing



$\left[\frac{-14}{9}, \infty\right)$

⑩ $4.9 + 2.6x < 4.2x - 4.7$

$-2.6x \quad -2.6x$ subtract 2.6x

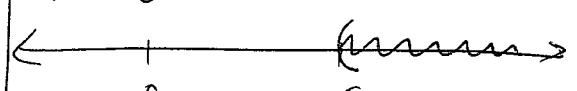
$4.9 < 1.6x - 4.7$

$+4.7 \quad +4.7$ add 4.7

$\frac{9.6}{1.6} < \frac{1.6x}{1.6}$ divide by 1.6

$6 < x$

$x > 6$



$(6, \infty)$